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Letter to the Editor

A Remark on Rational Approximation

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THEOREM. *Let $f(x)$ be a real function continuous on $[0, 1]$ with $f(1)=0$. Then for any real polynomials $P(x)$, $Q(x)$ which are $\neq 0$ on $[0, 1]$, we have*

$$\left\| f-\frac{P}{Q}\right\|_{L^\infty[0,1]} \geqslant \frac{f(0)}{[1+(P(0)\,Q(1))/(Q(0)\,P(1))]}.$$

Proof. Let

$$\left\| f-\frac{P}{Q}\right\|_{L^\infty[0,1]} = \varepsilon.$$

Then

$$\begin{aligned}\varepsilon \geqslant \frac{P(1)}{Q(1)} &= \left(\frac{P(0)}{Q(0)} - f(0)\right)\frac{Q(0)\,P(1)}{P(0)\,Q(1)} + \frac{f(0)\,Q(0)\,P(1)}{P(0)\,Q(1)}\\ &\geqslant -\frac{\varepsilon Q(0)\,P(1)}{P(0)\,Q(1)} + \frac{f(0)\,Q(0)\,P(1)}{P(0)\,Q(1)}\end{aligned}$$

and, so,

$$\varepsilon \geqslant \frac{f(0)}{[1+(P(0)\,Q(1))/(Q(0)\,P(1))]}.$$

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